

Roll No.

87031

**M. Sc. Mathematics 4th Semester
Under NEP - 2020 w. e. f. 2025-26
Examination – May, 2026**

INNER PRODUCT SPACES AND MEASURE THEORY

Paper : 25MAT204DS01

Time : Three Hours]

[Maximum Marks : 70

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question No. 1 is *compulsory*. All questions carry equal marks.

1. Compulsory questions :

$4 \times 3.5 = 14$

- (a) Show that inner product is sesquilinear.
- (b) Prove that every self-adjoint operator is normal.

- (c) Hahn Decomposition is not unique. Support this statement with an example.
- (d) What do you mean by conjugate of a Hilbert space?

SECTION – I

2. (a) Let X be an inner product space. Show that an inner product and the corresponding norm satisfy the Schwarz. Inequality and the triangle inequality. 10
- (b) State and prove Polarization Identity. 4
3. (a) Show that a closed convex subset C of a Hilbert space H contains a unique vector of smallest Norm. 7
- (b) State and prove Projection theorem. 7

SECTION – II

4. (a) Let H be a Hilbert space. Prove that H^* is also Hilbert space with respect to inner product defined as $(f_x, f_y) = (y, x)$. 7

- (b) Let T be a continuous operator on a Hilbert space H . Then there exists a unique continuous operators T^* such that 7

$$(Tx, y) = (x, T^*y), \quad \forall x, y \in H$$

5. (a) If A is a positive operator on a Hilbert space H . Then $I + A$ is non-singular. In particular, if T is an arbitrary operator on H then $I + T^*T$ and $I + TT^*$ are non-singular. 10

- (b) Show that the adjoint operation is one-one and onto as a mapping of $B(H)$ onto itself. 4

SECTION – III

6. (a) Prove that P is a projection on closed linear subspace M of H iff $I - P$ is projection on $M^\perp$. 7

- (b) If T is an operator on a Hilbert space H . Then T is normal iff its real and imaginary parts commutes. 7

7. (a) Let T be a normal operator on H . Prove that eigen spaces are pairwise orthogonal. 7

- (b) Show that a Hilbert space H is finite dimensional iff its every complete orthonormal subset is a basis for H . 7

SECTION – IV

8. (a) Show that every measurable subset of positive finite signed measure contains a positive set of positive signed measure. 10
- (b) Show that Hahn Decomposition is unique to the extent that $\{A_1, B_1\}$ and $\{A_2, B_2\}$ be the Hahn Decomposition with respect to ν such that $A_1 \Delta A_2$ is null set. 4
9. State and prove Radon-Nikodym theorem. 14
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Roll No.

87032

**M. Sc. Mathematics 4th Semester
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CLASSICAL MECHANICS

Paper : 25MAT204DS02

Time : Three Hours]

[Maximum Marks : 70

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question No. 1 is *compulsory*. All questions carry equal marks.

1. (a) Find M. I. of a rectangular lamina about a line through its centre and perpendicular to its plane. 4
- (b) Define Free-Canonical transformation and give an example. 3

(c) Derive the differential equation of rotation of a rigid body about a stationary axis. 4

(d) State and prove principle of least action for Lagrangian action. 3

SECTION - I

2. (a) Find the kinetic energy of a rigid body rotating about a fixed axis in terms of moments and products of inertia. 7

(b) Find the moment of inertia of a uniform triangular lamina about its base. 7

3. (a) Define equimomental systems and discuss necessary and sufficient conditions for them. 7

(b) Show that for a two-dimensional mass distribution, one of the principal axes at O is inclined at an angle θ to x -axis through O , where

$$\tan 2\theta = \frac{2F}{B - A}. \quad 7$$

SECTION – II

4. (a) Define possible velocity and possible acceleration.

7

- (b) Derive Lagrange's equations of the second kind. 7

5. (a) Define Lagrangian function and derive Lagrange's equations for potential forces. 7

- (b) The length of a plane simple pendulum changes with time such that $l = a + bt$, where a and b are constants. Find the Lagrange's equations of motion. 7

SECTION – III

6. (a) Derive Hamilton canonical equation and show

that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$. 7

- (b) State and prove Poisson's identity for Poisson brackets. 7

7. (a) State and prove Hamilton's Principle. 7

(b) Derive Jacobi equations. 7

SECTION – IV

8. (a) Define canonical transformation and derive the Hamilton-Jacobi equation. 7

(b) Prove that Lagrange's Bracket is invariant under a free univalent canonical transformation. 7

9. (a) Show that the transformation $Q = \log\left(\frac{1}{q} \sin p\right), P = q \cot p$ is canonical. 7

(b) State and prove Jacobi theorem. 7

Roll No.

87033

**M. Sc. Mathematics 4th Semester
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VISCOUS FLUID DYNAMICS

Paper : 25MAT204DS03

Time : Three Hours]

[Maximum Marks : 70

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting one question from each Section. Question No. 9 (Section - V) is compulsory. All questions carry equal marks.

SECTION – I

1. (a) Find the complex potential for vortices of strengths K_1 and K_2 situated at point a and b .

- (b) A vortex of circulation $2\pi k$ is at rest at the point $z = na(n > 1)$, in the presence of a plane circular impermeable boundary $|z| = a$ around which there is a circulation $2\pi\lambda B$. Show that $\lambda = \frac{1}{n^2 - 1}$.

Also find stagnation points.

2. Explain Isentropic Gas flow.

SECTION – II

3. Prove that stress matrix is symmetric.
4. Derive Navier-Stoke's equations for a viscous compressible fluid.

SECTION – III

5. Discuss the steady flow between two parallel plane for Couette's flow.

6. Explain the theory of very slow motion.

SECTION – IV

7. Explain the boundary layer equation in two-dimensions.

8. Discuss the momentum integral equation for the boundary layer.

SECTION – V

9. (a) Find image of a vortex in a plane.

(b) Define principle of superposition.

(c) What is Non-Newtonian fluid ?

(d) Define slipper block.

(e) What is error function ?

(f) What is Froude number ?

(g) Define dynamical similarity.

Roll No.

87034

M. Sc. Mathematics 4th Semester
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PARTIAL DIFFERENTIAL EQUATIONS

Paper : 25MAT204DS04

Time : Three Hours]

[Maximum Marks : 70

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting one question from each Section. Question No. 1 (Section - I) is compulsory.

SECTION – I

1. (a) What do you mean by steady state ?
- (b) What is the smoothing property of heat equation ?
- (c) Write down the statement of strong maximum Principle for the heat equation.

- (d) What do you mean by envelope ?
- (e) Define heat ball.
- (f) What is Legendre transform ?
- (g) What do you mean by spherical means ?

SECTION – II

- 2. Solve the heat equation $u_t = u_{xx}$ when the temperature at the end points are different.
- 3. (a) Solve the equation $u_{tt} = c^2(u_{xx} + v_{yy})$ when $0 < x < a, 0 < y < b$ with suitable initial and boundary condition.
(b) Find the solution of wave equation for infinite string.

SECTION – III

- 4. (a) Derive the solution of homogeneous transport equation.
(b) Find the fundamental solution of Laplace equation.

5. (a) Explain the concept of regularity for harmonic functions.

(b) Solve the boundary value problem $\Delta u = 0$ in $B(0,1)$
 $u = g$ on $\partial B(0,1)$

with the help of Green's function.

SECTION – IV

6. Find the solution of problem $u_t - \Delta u = 0$ in $R^n \times (0, \infty)$
 $u = g$ on $R^n \times \{t = 0\}$

associated with the homogeneous heat equation where

$$g \in C(R^n) \cap L^\infty(R^n).$$

7. (a) Find the solution of equation :

$$u_{tt} - u_{xx} = 0 \text{ in } R_+ \times (0, \infty)$$

$$u = g, u_t = h \text{ on } R_+ \times \{t = 0\}$$

where g and h are given with $u = 0$ on $\{x = 0\} \times (0, \infty)$
 $g(0) = 0, h(0) = 0$

(b) Prove that there exists almost one function $u \in C^2(\overline{U_T})$ which solves :

$$u_{tt} - \Delta u = f \text{ in } U_T$$

$$u = g; \text{ in } \Gamma_T$$

$$u_t = h \text{ on } U \times \{t = 0\}$$

SECTION - V

8. Explain the method of characteristics for solving

$$F(Du, u, x) = 0.$$

9. (a) State and prove convex duality of Hamiltonian and Lagrangian.

(b) Suppose $x \in \mathbb{R}^n$, $t > 0$ and u defined by the Hopf-Lax formula is differentiable at a point $(x, t) \in \mathbb{R}^n \times (0, \infty)$. Then $u_t(x, t) + H(Du(x, t)) = 0$

$$(0, \infty). \text{ Then } u_t(x, t) + H(Du(x, t)) = 0$$

Roll No.

87037

**M. Sc. Mathematics 4th Semester
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ARTIFICIAL INTELLIGENCE AND EXPERT SYSTEMS

Paper : 25MAT204DS07

Time : Three hours]

[Maximum Marks : 35

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt any *five* questions, selecting at least *one* from each Section. Question No 1 (Section-I) is *compulsory*. All questions carry equal marks.

SECTION – I

1. Attempt all parts :

- (a) Define Artificial Intelligence and mention its applications. 2
- (b) What is a state space ? Explain its role in problem solving. 2

(c) Define knowledge representation with an example. 2

(d) What is fuzzy logic ? Give one real-life example. 1

SECTION – II

2. (a) Explain problem formulation in Artificial Intelligence using state space representation. Illustrate with a suitable example showing initial state, goal state, and operators. 3.5

(b) Explain heuristic search and the role of heuristic functions in improving search efficiency. Provide suitable examples. 3.5

3. (a) Explain Alpha-Beta pruning in game playing with a detailed example and illustrate how branches are pruned. 3.5

(b) Explain the A * algorithm in detail including evaluation function. Demonstrate its working with an example. 3.5

SECTION – III

4. (a) Explain predicate logic representation and its importance in AI with suitable examples. 3.5

(b) Describe unification and resolution mechanisms used in logical inference. 3.5

5. (a) Compare semantic networks and frames highlighting their advantages and limitations. 3.5
- (b) Explain the process of Skolemization in predicate logic with a suitable example. 3.5

SECTION – IV

6. (a) Draw and explain the architecture of an Expert System, highlighting the role of the Knowledge Base and Inference Engine. 3.5
- (b) Discuss the different Knowledge Acquisition techniques used to gather expertise from human experts. 3.5
7. (a) Describe the various stages of the Expert System Development Life Cycle. 3.5
- (b) Explain the role of a Knowledge Engineer in the building of an expert system. 3.5

SECTION – V

8. (a) Explain the architecture and working of a Perceptron. What are its limitations? 3.5
- (b) Describe the Back-propagation learning algorithm. How does it help in training multi-layer networks? 3.5

9. (a) Discuss the components and working of a Fuzzy
Controller. 3.5

(b) List and describe practical applications of
Artificial Neural Networks in engineering or
mathematics. 3.5