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M. Sc. Mathematics 2nd Semester Under NEP-2020 w.e.f. 2024-25 (Fresh & Re-appear) Examination – May, 2026

THEORY OF FIELD EXTENSIONS

Paper : 24MAT202DS01

Time : Three hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 (Section-V) is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) Let $K|F$ be any field extension. Then, show that $a \in K$ is algebraic over F iff $[F(a):F]$ is finite and equals the degree of minimal polynomial of 'a' over F. 7

(b) Let K/F be any extension and $a \in K$ is algebraic over F. Let $p(x) \in F[x]$ be the minimal polynomial of 'a'. Then, show that $F[x]/<p(x)> \cong F[a] = F(a)$. 7

2. (a) Let $p(x) \in F[x]$ be an irreducible polynomial over F. Then, show that there always exists an extension E of F which contains atleast one root of $p(x)$ and $[E:F] = n = \deg p(x)$. 7

(b) Let $\eta : F \to F'$ be an isomorphism onto. Let $p(x)$ be any irreducible polynomial of degree n in $F[x]$ and $p'(t)$ be corresponding polynomial in $F'(t)$. Let u be any root of $p(x)$ and v be any root of $p'(t)$ in some extension of F and F' respectively. Then, show that there exists an isomorphism, say $\mu : F(u) \to F'(\nu)$ which is onto and is such that $\mu(\lambda) = \eta(\lambda)$ for all $\lambda \in F$ and $\mu(u) = \nu$. 7

SECTION – II

3. (a) Show that sin $m°$ is an algebraic integer for every integer m. 7

(b) Let ch.$F = p > 0$. Prove that the element 'a' in some extension of F is separable iff $F(a^p) = F(a)$. 7

4. (a) Let F be any field and K be its extension. Let $a \in K$ be algebraic over F of degree m and $b \in K$ be algebraic over F of degree n such that $(m, n) = 1$. Then, show that $[F(a,b):F] = mn$. 7

(b) Let $ch.F = p(\neq 0)$ and $f(x) \in F[x]$ be an irreducible polynomial over F. Then, prove that $f(x)$ is inseparable iff $f(x) \in F[x^p]$. 7

SECTION – III

5. Let G be a finite subgroup of $AutK$. If F_0 is fixed subfield under G, that is, $F_0 = \{x \in K : \sigma(x) = x \text{ for all } \sigma \in G\}$. Then, show that $[K : F_0] = O(G)$. 14

6. Let $K = F(\alpha)$ be a simple finite separable extension of F. Then, K is the splitting field of the minimal polynomial of α over F iff F is the fixed field under the group of all F-automorphisms of K. 14

SECTION – IV

7. Let K be a finite normal extension of F. If E be any subfield of K such that $F \subseteq E \subseteq K$, then show that K is normal extension of E. 14

8. Let K be a finite extension of F and ch.F = 0. Then, prove that K is normal extension of F iff the fixed field under $G(K, F)$ is F itself. 14

SECTION – V

9. (a) Show that every finite extension is an algebraic extension.

   (b) Let K/F be any extension and $f(x) \in F[x]$, then show that the element $a \in K$ is a root of polynomial $f(x)$ iff $(x-a) \mid f(x)$ in $K[x]$.

   (c) Let charF = 0 and $f(x)$ be any irreducible polynomial over F, then show that $f(x)$ cannot have multiple roots.

(d) Find the splitting field and its degree for the polynomial $f(x) = x^3 - 2$ over Q.

(e) Let $[K : F] = 2$, then show that K is normal extension of F always.

(f) If P is a prime subfield of K, then prove that $AutK = G(K, P)$, that is every automorphism on K fixes P.

(g) Define normal closure of an extension. $2 \times 7 = 14$

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M. Sc. Mathematics 2nd Semester Under NEP-2020 w.e.f. 2024-25 (Fresh & Re-appear) Examination – May, 2026

MEASURE AND INTEGRATION THEORY

Paper : 24MAT202DS02

Time : Three hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 (Unit-V) is ***Compulsory***. All questions carry equal marks.

UNIT – I

1. (a) Define and prove the countable subadditivity of Lebesgue measure.

(b) Let A be an algebra of subsets and $\{E_i : i \in N\}$ a sequence of sets in A. Then prove that there exists

a sequence $\{D_i : i \in N\}$ of disjoint members of A such that $D_i \subset E_i (i \in N)$ and $\bigcup_{i \in N} D_i = \bigcup_{i \in N} E_i$.

2. (a) Show that the interval (a, ∞) is measurable.

(b) If E_1 and E_2 are any two measurable sets, then show that :

$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2).$$

UNIT – II

3. (a) If α is an extended real number then show that the four conditions of measurable function imply that $\{x : f(x) = \alpha\}$ is also measurable.

(b) Show that characteristics function χ_A is measurable if and only if A is measurable.

4. State and Prove Lusin theorem.

UNIT – III

5. If f and g are bounded and measurable functions defined on a set E of finite measure, then prove that :

(i) $\int_E af = a \int_E f$

(ii) $\int_E (f+g) = \int_E f + \int_E g$

(iii) If $f \leq g$ on E, then $\int_E f \leq \int_E g$

(iv) If $f = g$ *a.e.* on E then $\int_E f = \int_E g$

(v) If $A \leq f(x) \leq B$, then $A.m(E) \leq \int_E f \leq B.m(E)$

(vi) If A and B are disjoint measurable sets of finite measure then :

$$\int_{A \cup B} f = \int_A f + \int_B f$$

6. (a) State and Prove Fatou's lemma.

(b) If f is integrable, then prove that $\left|\int_E f\right| \leq \int_E |f|$ and when does the equality holds.

UNIT – IV

7. (a) Prove that every function of bounded variation can be written as the difference of two increasing functions.

(b) If f is a function of bounded variation on [a, b], then show that f is measurable:

8. (a) If f is integrable on [a, b], and $\int_a^x f(t)dt = 0 \ \forall x \in [a,b]$, then show that $f(t) = 0$ *a.e.* on $[a, b]$.

(b) Prove that a function F is an indefinite integral if and only if F is an absolutely continuous function.

UNIT – V

9. (a) What is a null set ?

(b) Define convergence in measure.

(c) What is the difference between absolute and uniform continuity ?

(d) Define the total variation of a function.

(e) What is an indicator function ?

(f) What is the characteristics property of simple functions ?

(g) What is the difference between simple and step functions ?

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M. Sc. (Mathematics) 2nd Semester Under NEP-2020 w. e. f. 2024-25 (Fresh & Fe-appear) Examination – May, 2026

INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS

Paper : 24MAT202DS03

Time : Three Hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** question in all, selecting ***one*** questions from each Section. Question No. **9** (Section-**V**) is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) Reduce the initial value problem $y''-3y'+2y=4\sin x,\ y(0)=1,\ y'(0)=-2$ to a Volterra integral equation.

(b) Discuss the method of successive approximation to solve Volterra integral equation of second kind.

2. (a) Reduce the integral equation $\sin x = \lambda \int_0^x e^{x-\xi} u(\xi)\, d\xi$ to the second kind and find solution of it.

(b) Using the resolvent Kernel method, solve the integral equation $u(x) = x + \lambda \int_0^x e^{2x-\xi} u(\xi) d\xi$.

SECTION – II

3. (a) Explain the method of successive substitution to solve Fredholm integral equation of second kind.

(b) Find the resolvent Kernel of the below integral equation and find solution for it :

$$u(x) = x + \lambda \int_0^2 (1 - 3x\xi)\, u(\xi)\, d\xi.$$

4. (a) Find the eigen values and eigen function of the integral equation $u(x) = \lambda \int_0^{2\pi} \sin(x+t)\, u(t)\, dt$.

(b) Solve the integral equation by the method of degenerate Kernel $u(x) = x + \lambda \int_0^1 (1 + x + t) u(t)\, dt$.

SECTION – III

5. (a) Construct Green's function for the equaiton $xu'' + u' = 0$ with the condition u(x) is bounded as $x \to \theta$, $u(1) = \mu u'(1)$, $\mu \neq 0$.

(b) Using Green's function, solve the B.V.P. $u'' + u = x$ with the condition $u(0) = 0$ and $u\left(\frac{\pi}{2}\right) = 0$.

6. Explain the construction of Green's function by variation of parameter method.

SECTION – IV

7. (a) If $\alpha(x)$ and $\beta(x)$ are continuous in $[a, b]$ and if $\int_a^b [\alpha(x)h(x) + \beta(x)h'(x)]dx = 0$ for every function $h(x) \in D_1[a, b]$ such that $h(a) = h(b) = 0$, then prove that $\beta(x)$ is differentiable and $\beta'(x) = \alpha(x)$ for all x in $[a, b]$.

(b) Derive euler's equation when functionals dependent on higher order derivatives.

8. (a) Determine the surface of least area spanned by a given contour.

(b) Find the extremals of the functional $\int_0^{\theta_1} r\sin\theta\sqrt{r^2+r'^2}\,d\theta$ using the transformation $x = r\cos\theta,\ \ y = r\sin\theta$.

SECTION – V

9. (a) Define variable and point problem.

(b) What do you mean by strong extremum ?

(c) What is self-adjoint operator ?

(d) What do you mean by symmetric property of Green's function ?

(e) Define separable Kernel.

(f) What is the difference between successive approximation and substitution for solving integral equation ?

(g) Solve the integral equation $\sin x = \lambda \int_0^x e^{x-\xi} u(\xi)\,d\xi$

(h) Define Volterra integral equation of second kind.

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M. Sc. (Mathematics) 2nd Semester
Under NEP - 2020 w. e. f. 2024-25
(Fresh & Re-appear)
Examination – May, 2026

OPERATIONS RESEARCH TECHNIQUES

Paper : 24MAT202DS04

Time : Three Hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section (**I, II, III** and **IV**) and Question No. **1** is ***compulsory***. All questions carry equal marks.

1. (a) Give mathematical formulation of a general linear programming problem. 2

(b) What do you mean by 'Slack' and 'surplus' variables in linear programming ? 1½

(c) Mention the common features of transportation problems and assignment problems. 2

(d) State how will you handle an unbalanced assignment problem ? 1½

(e) In queueing theory, what is traffic intensity ? Give its significance. 2

(f) Define a 'Stochastic process'. 1½

(g) Distinguish between 'Time Horizon' and 'Lead Time' in the context of inventory analysis. 2

(h) Write a short note on 'Two person-zero-sum game'. 1½

SECTION – I

2. (a) What do you mean by 'Operations Research' ? Give its scope. 8

(b) Explain graphical method to solve a linear programming problem. 6

3. (a) Solve the following linear programming problem using Big-method : 10

Maximize $Z = 5x_1 - 2x_2 + 3x_3$

subject to

$$2x_1 + 2x_2 - x_3 \geq 2$$
$$3x_1 - 4x_2 \leq 3$$
$$x_2 + 3x_3 \leq 5$$

and $x_1, x_2, x_3 \geq 0$.

(b) Discuss the 'Concept of duality' in linear programming. 4

SECTION – II

4. (a) What is a transportation problem ? Explain its types giving suitable examples. 5

(b) Solve the following assignment problem : 9

		Machines			
		M_1	M_2	M_3	M_4
Jobs	J_1	5	7	11	6
	J_2	8	5	9	6
	J_3	4	7	10	7
	J_4	10	4	8	3

5. (a) Using 'Modified Distribution Method', solve the following transpiration problem : 9

		Destination				Supply
		D_1	D_2	D_3	D_4	
Origin	O_1	1	2	1	4	30
	O_2	3	3	2	1	50
	O_3	4	2	5	9	20
Demand		20	40	30	10	100

(b) Write a short note on 'Travelling salesman problem.' 5

SECTION – III

6. (a) In the context of queueing theory, explain the following : 6

(i) Poisson process

(ii) Arrival Pattern

(iii) Queue discipline.

(b) Discuss M/M/1 queuering model and its important characteristics. 8

7. (a) Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 3 trains, find : 6

(i) The probability that the yard is empty,

(ii) The average number of trains at the yard,

(iii) The average waiting time of a new train coming in to the yard.

(b) What are multiple server queueing models ? Explain any *one* of them in detail. 8

SECTION – IV

8. (a) Discuss EOQ model when shortages are allowed (instantaneous supply case). 8

(b) Distinguish between the following : 6

(i) Pure and mixed strategic.

(ii) Saddle point and value of the game.

(iii) Determinable and fair games.

9. (a) What do you mean by inventory control ? Give its objectives. 6

(b) Solve the game whose pay-off matrix is given under : 8

$$\begin{bmatrix} 6 & 2 & 4 & 8 \\ 2 & -1 & 1 & 12 \\ 2 & 3 & 3 & 9 \\ 5 & 2 & 6 & 10 \end{bmatrix}.$$

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M. Sc. Mathematics 2nd Semester Under NEP-2020 w.e.f. 2024-25 (Fresh & Re-appear) Examination – May, 2026

ALGEBRAIC NUMBER THEORY

Paper : 24MAT202DS05

Time : Three hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Unit. Question No. **1** is ***compulsory***. All questions carry equal marks.

1. (a) Show that if $|N(\alpha)| = 1$, then α must be unity. 2

(b) Is there any field in which fundamental theorem false ? If yes, then explain. 2

(c) Show that the set of algebraic numbers is countable. 2

(d) Let α be an algebraic number show that there exists $m \in Z$ such that $m\alpha$ is an algebraic integer. 2

(e) Let $K = Q(i)$, show that $i \in O_K$ and verify that $N_K(i)$ and $T_K(i)$ are integers. 2

(f) Define discriminant of an algebraic number field. 2

(g) State the Chinese Remanider theorem. 1

(h) State Dedekind's theorem. 1

UNIT – I

2. (a) State and prove fundamental theorem in $Q(w)$. 7

(b) If K is a quadratic field then there exists a non-zero square free integer m such that $K = Q(\sqrt{m})$, $m \neq 0, 1$. 7

3. (a) Show that the number of real Euclidean fields $Q(\sqrt{m})$ where $m \equiv 2$ or $3 \pmod 4$ is finite. 7

(b) State and prove Fermat theorem in the ring of Gaussian integers. 7

UNIT – II

4. (a) Let α be an algebraic number, there exists a unique polynomial $p(x)$ in $Q[x]$ which is monic, irreducible and of smallest degree, such that $p(\alpha) = 0$. Furthermore, if $f(x)$ in $Q[x]$ and $f(\alpha) = 0$, then $p(x)/f(x)$. 7

(b) Show that $\sum 2^{-3^n}$ is transcendental. 7

5. (a) State and prove theorem of primitive elements. 7

(b) Let $K = Q(\theta)$ be of degree n over Q. Let $w_1\ w_2$ ..... w_n be a basis of K as a vector space over Q. Show that the matrix $\Omega = \left[w_i^{(j)} \right]$ is invertible. 7

UNIT – III

6. (a) The bilinear pairing given by $B(x, y) : K \times K \to Q$ such that $(x, y) \to T_K(xy)$ is non -degenerate. 7

(b) Show that there exists $w_1^* w_2^* ..... w_n^* \in K$ such that $O_K \subseteq Zw_1^* + Zw_2^* + ... + Zw_n^*$. 7

7. (a) Let $K = Q\left(\sqrt{D}\right)$ with D a square free integer, find an integral basis for O_K. 7

(b) If $Q \subseteq K \subseteq L$ and K, L are algebraic number fields, show that d_K / d_L. 7

UNIT – IV

8. Define Dedekind domain. Show that O_K is Dedekind domain. 14

9. (a) Show that : 7

$$\gcd(A, B) = A + B = \prod_{i=1}^{r} P_i^{\min(e_i, f_i)} .$$

(b) Show that : 7

$$\text{ord}_p(AB) = \text{ord}_p(A)\text{ord}_p(B)$$
